

MULTI-OBJECTIVE STOCHASTIC INTUTIONISTIC FUZZY LINEAR PROGRAMMING PROBLEM

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ABSTRACT

Stochastic or random programming (SP) is a framework for modeling Linear Programming Problems (LPP) that fasten haziness. The common goal is tattered in solving random LPPs has still been to alter a stochastic model into the deterministic model and is achievable when the RHS constraint follows some explicit distributions. Here, a multi-objective stochastic programming problem (MOSPP) has been well thought-out with RHS constraint following Power Function distributions (PFD) of $F(\tilde{k}_i) = 1 - \tilde{Q}_i e^{-\tilde{R}_i h(\tilde{k}_i)}$. In this advance, the MOSPP is then solved by the simplex method. Mathematical examples are offered to demonstrate the proposed approach.

KEYWORDS: Multi-objective stochastic linear programming (MOSLP), Power function distributions & Heptagonal Intuitionistic Fuzzy Number (HptIFN)

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1. INTRODUCTION

One of the widespread problems in the sensible purpose of mathematical programming (MP) is the complexity within shaping the appropriate ethics of model parameters. The ethics of these parameters are habitually predisposed by random events that are unfeasible to forecast i.e., some or all of the replica parameters may be haphazard variables. What is desirable is a mode to put together a problem so that the optimization will directly deem the insecurity. One such approach for MP under vagueness is SP. The SP is an optimization system in which the constraints and/or the objective function of an optimization problem contains certain subjective variables. Model coefficients of most of these models are implicit to trail self-governing normal distribution because deriving the deterministic equivalent of the objective function and/or constraints of the replica is well known (Kall and Wallace, 1994)[9] in this case. SP models were first formulated by Dantzig (1955)[7] who suggested a two-stage programming performance that involves alteration of SP models into their equivalent deterministic programming (DP) models. However, this system suffers from the control that it does not allow any constraint to be dishonored even at definite probability level. The properties of SP problems and methods for obtaining optimal solution have been described in Rao (1989) [11], Kall and Wallace (1994)[9], Birge and Louveaux (1997)[4] and Pre'kopa (1995)[10]. In the recent precedent, SP has been functional to the problems having numerous, incompatible and non-commensurable objectives where generally there does not survive alone result which can maximize (minimize) all the objectives. However, in a various criteria decision-making system, the decision-maker normally follows on fulfillment of criteria moderately than maximization (minimization) of objectives. Numerous methods for solving MOSLPPs have been developed by Leclercq (1982), Goicoechea et al. (1982)[8]. Baba and

Morimoto (1993)[3] proposed a stochastic approximation method for solving the MOSLPP and Caballero et al. (2001)[5] provided resourceful solution concepts in MOSPP. Suwarna et al. (1997)[14] renewed MOSLPP into a DP. Abdelaziz et al. (2007)[2] presented multi-objective programming techniques to choose the group best satisfying the decision makers' aspirations and preferences. Most of the probabilistic models assume normal distribution for model coefficients. Sahoo and Biswal (2005)[13] offered various deterministic equivalents for the probabilistic disaster connecting normal and log-normal indiscriminate variables for mutual constraints. However, most of the papers be unsuccessful to deal with more than two distributions, moreover we cannot only transmit on normal, lognormal and exponential distributions when we deal with realtime empirical modeling. Hence, this article addresses the PFD, wherein one can sculpt significant submission when the essentials of source vector follow the distribution. In this paper, we have converted the MOSLPP model into DP model, where RHS follows the PFD.

2. PRELIMINARIES

2.1 Heptagonal Intuitionistic Fuzzy Number [12]

“A Heptagonal intuitionistic fuzzy number is specified by $\tilde{A}_{Hp}^I = (p_1, p_2, p_3, p_4, p_5, p_6, p_7)(p'_1, p'_2, p'_3, p'_4, p'_5, p'_6, p'_7)$ where $p_1, p_2, p_3, p_4, p_5, p_6, p_7, p'_1, p'_2, p'_3, p'_4, p'_5, p'_6, p'_7$ are real numbers such that $p'_1 \leq p_1 \leq p'_2 \leq p_2 \leq p'_3 \leq p_3 \leq p'_4 \leq p_4 \leq p'_5 \leq p_5 \leq p'_6 \leq p_6 \leq p'_7 \leq p_7$ and its membership and non membership are given by

$$\mu_{\tilde{A}_{Hp}^I}(x) = \begin{cases} 0 & \text{for } x < p_1 \\ \frac{1}{2} \left(\frac{x - p_1}{p_2 - p_1} \right) & \text{for } p_1 \leq x \leq p_2 \\ \frac{1}{2} & \text{for } p_2 \leq x \leq p_3 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{x - p_2}{p_3 - p_2} \right) & \text{for } p_3 \leq x \leq p_4 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{p_5 - x}{p_5 - p_4} \right) & \text{for } p_4 \leq x \leq p_5 \\ \frac{1}{2} & \text{for } p_5 \leq x \leq p_6 \\ \frac{1}{2} \left(\frac{p_7 - x}{p_7 - p_6} \right) & \text{for } p_6 \leq x \leq p_7 \\ 0, & \text{otherwise} \end{cases}$$

$$\gamma_{\tilde{A}_{Hp}^I}(x) = \begin{cases} 1 & \text{for } x < p'_1 \\ 1 - \frac{1}{2} \left(\frac{x - p'_1}{p'_2 - p'_1} \right) & \text{for } p'_1 \leq x \leq p'_2 \\ \frac{1}{2} & \text{for } p'_2 \leq x \leq p'_3 \\ \frac{1}{2} \left(\frac{x - p'_3}{p'_4 - p'_3} \right) & \text{for } p'_3 \leq x \leq p'_4 \\ 0 & \text{for } x = p'_4 \\ \frac{1}{2} \left(\frac{p'_5 - x}{p'_5 - p'_4} \right) & \text{for } p'_4 \leq x \leq p'_5 \\ \frac{1}{2} & \text{for } p'_5 \leq x \leq p'_6 \\ 1 - \frac{1}{2} \left(\frac{p'_7 - x}{p'_7 - p'_6} \right) & \text{for } p'_6 \leq x \leq p'_7 \\ 1, & \text{otherwise} \end{cases} ,$$

2.2 Value of Heptagonal Intuitionistic Fuzzy Number

Let \tilde{p}_α and \tilde{p}_β be any α cut and β cut set of an HptIFN \tilde{A}_{Hp}^I respectively. The membership function and non-membership function values of $\mu_{\tilde{A}_{Hp}^I}(x)$ and $\gamma_{\tilde{A}_{Hp}^I}(x)$ for the HptIFN \tilde{A}_{Hp}^I is defined as

$$V_\mu = \int_0^1 (L_{\tilde{A}_{Hp}^I}(\alpha) + R_{\tilde{A}_{Hp}^I}(\alpha)) f(\alpha) d\alpha \quad (2.1)$$

$$V_\gamma = \int_0^1 (L_{\tilde{A}_{Hp}^I}(\beta) + R_{\tilde{A}_{Hp}^I}(\beta)) g(\beta) d\beta \quad (2.2)$$

respectively.

The function $f(\alpha) = \alpha$ and $g(\beta) = 1 - \beta$ gives different mass to elements in different α and β cut sets. In fact, α vanishes the donation of the lower α -cut sets and (β) vanishes the contribution of the higher β -cut sets, which is sensible since these cut sets arising from values of $\mu_{\tilde{A}_{Hp}^I}(x)$ and $\gamma_{\tilde{A}_{Hp}^I}(x)$ have a significant quantity of hesitation. Obviously, $V_\mu(\tilde{A}_{Hp}^I)$ and $V_\gamma(\tilde{A}_{Hp}^I)$ reflects the information on every membership and non-membership degree and may be regarded as a central value that represents from the membership and non-membership function point of view. According to (2.1), the value of the membership function of a HptIFN \tilde{p} is calculated as follows:

$$L_{\tilde{A}_{Hp}^I}(\alpha) = 2\alpha(p_2 - p_1) + p_1 \text{ and } R_{\tilde{A}_{Hp}^I}(\alpha) = p_7 - 2\alpha(p_7 - p_6)$$

$$\text{Substituting } L_{\tilde{A}_{Hp}^I}(\alpha) \text{ and } R_{\tilde{A}_{Hp}^I}(\alpha) \text{ in (2.1) we get } V_\mu = \frac{4p_2 - p_1 - p_7 + 4p_6}{6} \quad (2.3)$$

In a similar way, according to (2.2), the value of the non-membership function of a HptIFN \tilde{p} is calculated as follows:

$$L_{\tilde{A}_{Hp}^I}(\beta) = 2(1-\beta)(p'_2 - p'_1) + p'_1 \text{ and } R_{\tilde{A}_{Hp}^I}(\beta) = p'_7 - 2(1-\beta)(p'_7 - p'_6)$$

$$\text{Substituting } L_{\tilde{A}_{Hp}^I}(\beta) \text{ and } R_{\tilde{A}_{Hp}^I}(\beta) \text{ in (2.2) we get } V_\gamma = \frac{4p'_2 - p'_1 - p'_7 + 4p'_6}{6} \quad (2.4)$$

2.3 The Value Based Ranking Method

By the above arrived value of a HptIFN, a new ranking method of HptIFNs is projected in this clause.

Let $\tilde{A}_{Hp}^I = (p_1, p_2, p_3, p_4, p_5, p_6, p_7)(p'_1, p'_2, p'_3, p'_4, p'_5, p'_6, p'_7)$ be a HptIFN. A value-index for \tilde{p} are defined as:

$$V_\theta(\tilde{a}) = \theta V_\mu(\tilde{p}) + (1-\theta)V_\gamma(\tilde{p}) \quad (2.5)$$

respectively, where $\theta \in (0,1)$ is a mass which represents the decision maker's preference information. $\theta \in (0.5, 1]$ shows that the decision maker prefers the feeling of ambiguity; $\theta \in [0, 0.5)$ shows that the decision makers feeling is positive; $\theta = 0.5$ shows that the decision maker is in confused state. Therefore, the value index may reflect the decision maker's subjectivity attitude to the HptIFNs.

3. MOSIFLPP WITH MARGINAL CONSTRAINT FOR GENERAL FORM OF DISTRIBUTIONS [6]

The mathematical model of a MOSLPP can be given as:

$$\max \tilde{z}_k = \sum_{j=1}^n \tilde{c}_j^k \tilde{x}_j, \text{ Where } k = 1, 2, 3, \dots, K \quad (3.1)$$

$$\text{Subject to } P\left(\sum_{i=1}^n \tilde{a}_{ij} \tilde{x}_j \leq \tilde{k}_i\right) \geq \tilde{p}_i, i = 1, 2, 3, \dots, m \quad (3.2)$$

$$\tilde{x}_j \geq 0, j = 1, 2, 3, \dots, n.$$

where $\tilde{c}_j^k, \tilde{a}_{ij}, \tilde{k}_i$ are fuzzy numbers, \tilde{x}_j are fuzzy variables, $0 < \tilde{p}_i < 1$ and more or less equal to one. We assume that the parameters \tilde{a}_{ij} and \tilde{c}_j^k are deterministic constants and \tilde{k}_i are random variables having PFD $F(\tilde{k}_i) = 1 - \tilde{Q}_i e^{-\tilde{R}_i h(\tilde{k}_i)}$. It is also consider that in the i^{th} random variable k_i has two known parameters namely $\tilde{R}_i (\neq 0)$ and $\tilde{Q}_i (> 0)$, where \tilde{R}_i and \tilde{Q}_i are such that $F(\alpha_i) = 0, F(\beta_i) = 1$ and $h(\tilde{k}_i)$ is a monotonic and differentiable function of \tilde{k}_i in the interval (α_i, β_i) . In this model, the decision variables $\tilde{x}_j, j = 1, 2, 3, \dots, n$. are treated as deterministic decision variables. The probability density function of the random variable \tilde{k}_i is given by

$$f(\tilde{k}_i) = \tilde{R}_i \tilde{Q}_i e^{-\tilde{R}_i h(\tilde{k}_i)} h'(\tilde{k}_i) \quad (3.3)$$

Equation (3.2) can be uttered as

$$P(\tilde{k}_i \geq \tilde{y}_i) \geq \tilde{p}_i, i = 1, 2, 3, \dots, m \quad (3.4)$$

$$\text{where } \tilde{y}_i = \sum_{j=1}^n \tilde{a}_{ij} \tilde{x}_j$$

equation (3.3) can be redefined as

$$\int_{\tilde{y}_i}^{\beta_i} \tilde{R}_i \tilde{Q}_i e^{-\tilde{R}_i h(\tilde{k}_i)} h'(\tilde{k}_i) d\tilde{k}_i \geq \tilde{p}_i, i = 1, 2, \dots, m$$

After the process of integration we have

$$\tilde{Q}_i e^{-\tilde{R}_i h(\tilde{y}_i)} \geq \tilde{p}_i, \text{ as } \tilde{Q}_i e^{-\tilde{R}_i h(\beta_i)} = 0 \quad (3.5)$$

When \tilde{k}_i follow PFD

The PFD is given by

$$F(\tilde{k}_i) = \lambda_i^{-\tilde{a}_i} \tilde{b}_i^{\tilde{a}_i}, 0 \leq \tilde{k}_i \leq \lambda_i, \tilde{a}_i > 0, \lambda_i > 0 \quad (3.6)$$

Here $\tilde{R}_i = -1, \tilde{Q}_i = 1, h(\tilde{k}_i) = \log(1 - \lambda_i^{-\tilde{a}_i} \tilde{k}_i^{\tilde{a}_i})$ now from (5), we have

$$e^{\log(1 - \lambda_i^{-\tilde{a}_i} \tilde{y}_i^{\tilde{a}_i})} \leq \tilde{p}_i \text{ this can be simplified as}$$

$$\tilde{y}_i \leq \lambda_i (1 - \tilde{p}_i)^{1/\tilde{a}_i} \quad (3.7)$$

So, the multi-objective intuitionistic fuzzy deterministic mathematical model can be uttered as

$$\left. \begin{array}{l} \max \tilde{z}_k = \sum_{j=1}^n \tilde{c}_j^k \tilde{x}_j, \text{ Where } k = 1, 2, 3, \dots, K \\ \text{Subject to } \sum_{j=1}^n \tilde{a}_{ij} \tilde{x}_j \leq \lambda_i (1 - \tilde{p}_i)^{1/\tilde{a}_i}, i = 1, 2, \dots, m \\ \text{and } \tilde{x}_j \geq 0, j = 1, 2, 3, \dots, n. \end{array} \right\} \quad (3.8)$$

4. ALGORITHM

The subsequent ladder to be followed to work out the LPP

Step 1: Formulate the given problem into LPP and convert into MOSIFLPP.

Step 2: Defuzzify the HpIFLPP into crisp LPP using the above value based ranking function.

$$V_\theta(\tilde{a}) = \theta V_\mu(\tilde{a}) + (1 - \theta) V_\gamma(\tilde{a})$$

Step 3: Converting the multi-objective HpIFLPP into a single objective HpIFLPP by taking average for the cost and considering the value of $\lambda = 10$ and $a = 5$.

Step 4: Solving the above crisp LPP of Step 3 to get the optimum solution for different θ values.

5. NUMERICAL EXAMPLE

A farmer is to grow carrot, radish, and cabbage in a season in areas be $\tilde{x}_j, j = 1, 2, 3$. (unit 10 acres = 1000m²) respectively. The probability of labor work time available for experienced, trainer and the non-experienced are 0.98, 0.95 and 0.90 hours. The profit coefficient and work time for the crops are given in table-1

Table 1

	Carrot	Radish	Cabbage
C ₁	5	6	3
C ₂	6	3	5
C ₃	2	5	8
Work time - Experienced	2	8	5
Work time – trainer	5	3	2
Work time - non Experienced	3	2	2

Solution:

Step 1: Since the profit from each crop and the time availability are uncertain, the number of units to be produced on each crop will also be uncertain. So we will model the problem as an intuitionistic fuzzy linear programming problem and use HpIFNs for each uncertain value.

Profit for C₁₁ which is close to 5 is modelled as $[-1, 1, 3, 5, 7, 9, 11][-3, -1, 1, 3, 5, 7, 9]$. Similarly, the other parameters are also modeled as HpIFNs taking into consideration the nature of the problem and other requirements. So a stochastic multi-objective heptagonal intuitionistic fuzzy LPP is formulated as.

$$\max z_1 = 5\tilde{x}_1 + 6\tilde{x}_2 + 3\tilde{x}_3, \quad \max z_2 = 6\tilde{x}_1 + 3\tilde{x}_2 + 5\tilde{x}_3, \quad \max z_3 = 2\tilde{x}_1 + 5\tilde{x}_2 + 8\tilde{x}_3$$

$$P(2\tilde{x}_1 + 8\tilde{x}_2 + 5\tilde{x}_3 \leq \tilde{b}_2) \leq 0.98$$

$$\text{Subject to, } P(5\tilde{x}_1 + 3\tilde{x}_2 + 2\tilde{x}_3 \leq \tilde{b}_3) \leq 0.95 \text{ and } \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq \tilde{0}$$

$$P(3\tilde{x}_1 + 2\tilde{x}_2 + 2\tilde{x}_3 \leq \tilde{b}_1) \leq 0.90$$

Step 2: Defuzzify the HpIFLPP into crisp LPP using the above value-based ranking function (2.5)

$$\max z_1 = (5\theta + (1-\theta)3)\tilde{x}_1 + (6\theta + (1-\theta)4)\tilde{x}_2 + (3\theta + (1-\theta))\tilde{x}_3,$$

$$\max z_2 = (6\theta + (1-\theta)4)\tilde{x}_1 + (3\theta + (1-\theta))\tilde{x}_2 + (5\theta + (1-\theta)3)\tilde{x}_3,$$

$$\max z_3 = (2\theta)\tilde{x}_1 + (5\theta + (1-\theta)3)\tilde{x}_2 + (8\theta + (1-\theta)6)\tilde{x}_3$$

Subject to,

$$P((2\theta)\tilde{x}_1 + (8\theta + (1-\theta)6)\tilde{x}_2 + (5\theta + (1-\theta)3)\tilde{x}_3 \leq \tilde{b}_2) \leq (0.98\theta + (1-\theta)0.78)$$

$$P((5\theta + (1-\theta)3)\tilde{x}_1 + (3\theta + (1-\theta))\tilde{x}_2 + (2\theta)\tilde{x}_3 \leq \tilde{b}_3) \leq (0.95\theta + (1-\theta)0.75)$$

$$P((3\theta + (1-\theta))\tilde{x}_1 + (2\theta)\tilde{x}_2 + (2\theta)\tilde{x}_3 \leq \tilde{b}_1) \leq (0.9\theta + (1-\theta)0.7)$$

$$\text{and } \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq \tilde{0}$$

Step 3: Converting the multi-objective HpIFLPP into a single objective HpIFLPP by taking an average for the cost and considering the value of $\lambda=10$ and $a=5$. Then the LPP is

$$\max Z = \left(\frac{13\theta + (1-\theta)7}{3} \right) \tilde{x}_1 + \left(\frac{14\theta + (1-\theta)8}{3} \right) \tilde{x}_2 + \left(\frac{16\theta + (1-\theta)10}{3} \right) \tilde{x}_3,$$

Subject to,

$$(2\theta)\tilde{x}_1 + (8\theta + (1-\theta)6)\tilde{x}_2 + (5\theta + (1-\theta)3)\tilde{x}_3 \leq 10(1 - (0.98\theta + (1-\theta)0.78))^{1/5}$$

$$(5\theta + (1-\theta)3)\tilde{x}_1 + (3\theta + (1-\theta))\tilde{x}_2 + (2\theta)\tilde{x}_3 \leq 10(1 - (0.95\theta + (1-\theta)0.75))^{1/5}$$

$$(3\theta + (1-\theta))\tilde{x}_1 + (2\theta)\tilde{x}_2 + (2\theta)\tilde{x}_3 \leq 10(1 - (0.9\theta + (1-\theta)0.7))^{1/5}$$

$$\text{and } \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq \tilde{0}$$

Step 4: Solving the above crisp LPP of Step 3 to get the optimum solution for $\theta = 0, 0.5$ and 1 .

Table 2

θ values	Max Z	X_1	X_2	X_3
0	10	2.53	0	2.46
0.5	10.20	1.39	0	1.29
1	6.70	0.83	0	0.58

6. CONCLUSIONS

The deterministic constraints (3.3) for the agreed probabilistic constraints where b_i 's pursue PFD for self-determining constraints are studied. With the help of (3.3), one can effortlessly acquire the deterministic MOLPP to the given MOSIFLPP with self-governing constraints. The main contribution of this paper is the derivation of deterministic correspondence for self-determining constraints where the RHS constraint following PFD by taking the suitable value of \tilde{R}_i , \tilde{Q}_i and $h(\tilde{k}_i)$. After the translation of MOSIFLPP into MOIFLPP, the ensuing MOLPP is then renewed into single objective LPP by using middling and then it is solved by simplex. A mathematical problem as an example is presented to show the success of the suggested method.

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